A Comparative Phase Plane Analysis of the FitzHugh–Nagumo and Izhikevich Models

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1 Introduction

Computational neuroscience utilises reduced mathematical models to capture the essential non-linear dynamics of neuronal activity. While the biophysical processes of the brain are high-dimensional, simplified models allow for a tractable investigation of excitability and spike generation through phase plane analysis (Cessac and Samuelides, 2007). This study compares the FitzHugh–Nagumo (FHN) and Izhikevich models under three stimulation protocols: stable current, single pulses, and double-pulsed current (Musacchio, 2025). Both models represent simplified versions of the biological neuron, reducing neuronal dynamics to two-dimensional systems, but differ significantly in their mathematical principles and approaches to spike generation and recovery dynamics. The two models have never been compared, which is a research gap this experiment aims to bridge to show the possibilities of use of the models.

2 Methods

2.1 Models

The FitzHugh–Nagumo model describes neuronal activity using a fast voltage variable and a slower recovery variable, making the system suitable for direct qualitative analysis through nullclines and phase plane trajectories (Cessac and Samuelides, 2007). The Izhikevich model similarly reduces neuronal behaviour to two ordinary differential equations, but introduces an explicit reset condition responsible for spike generation rather than continuous nonlinear evolution (Cessac and Samuelides, 2007; Izhikevich, 2003).

2.2 Current Experiments

Both models were introduced to varying levels of injected current, and their behaviour was observed under stable, single-pulsed, and double-pulsed stimulation (Musacchio, 2025).

3 Results

Experiments with stable and pulsed external current showed that the FitzHugh–Nagumo model reproduces sub-threshold oscillations, action potentials, spike trains, and both absolute and relative refractory periods. Furthermore, the timing of pulsed stimulation strongly influenced whether the system generated an action potential, confirming that excitability depends on the trajectory of the dynamical system.

Under constant and pulsed external currents, the Izhikevich model demonstrated excitability thresholds and tonic spiking behaviour analogous to biological neurons. However, the neuron was unable to sustain an absolute refractory period when the sustained stable external current introduced was too high, as the model has a hard reset value, which overrides it. The phase plane analysis revealed that spike generation depends much more on the magnitude of the input current than on its timing, as opposed to the FitzHugh–Nagumo model.

During the majority of conditions, the two models performed very similarly, with differences emerging primarily under extreme conditions.

4 Conclusion

The FitzHugh–Nagumo and Izhikevich models provide two different approaches to computationally modelling a neuron. Both qualitatively capture essential neuronal properties, yet offer different analytical advantages. The FHN model remains a superior tool for studying the continuous geometry of the action potential and bifurcation behaviour, while the Izhikevich model provides a flexible and efficient framework for simulating complex spike trains and tonic behaviour. The results provide valuable insight into choosing which model to use based on the desired experiment.

References

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